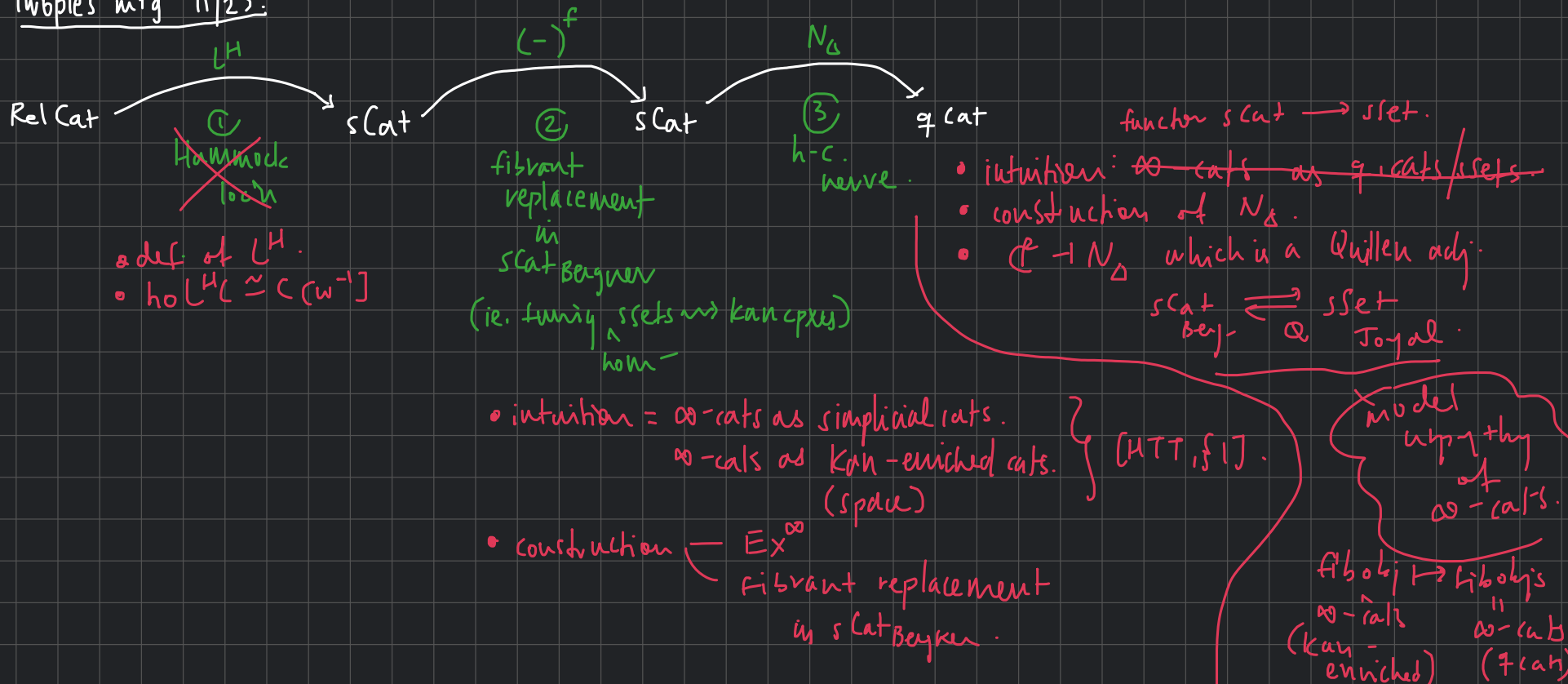


Twoples mtg 11/23.



(4) (if time) then: Quillen eqs \rightarrow eqs of ∞ -cats.

Q: In what sense does $N_\Delta C$ rep. the same htpy thg as C as a sCat?

(A) there's a htpy cat. functor $sCat \xrightarrow{ho_s} Cat$

$C \xrightarrow{ho_s} ho_s C = ob = ob(C)$
 $Hom_{ho_s C}(x,y) = \Pi_0 Hom_C(x,y)$

this agrees w/ the "usual" htpy cat. of gcats.

$gCat \xrightarrow{h} Cat$
 $X \xrightarrow{h} hX = ob = X_0$
 $Hom_{hX}(x,y) = X_1 / \sim_0$

Turn out that these agree

$sCat \xrightarrow{N_\Delta} gCat$
 $ho_s \searrow \quad \swarrow h$
 Cat

commutes.

(see here, 3.10)

[B].

Q: Ex^∞ vs. fibrant replacement?

(A): What is the model structure in question? — $s\text{Cat}_{\text{Bergner}}$ — we: "D-K equivalences"

$f: \mathcal{C} \rightarrow \mathcal{D}$ in $s\text{Cat}$ s.t.

- $f_{x,y}: \text{Hom}_{\mathcal{C}}(x,y) \rightarrow \text{Hom}_{\mathcal{D}}(fx, fy)$ is a w.e. of ssets. ??
- $\text{hof}: \text{ho}\mathcal{C} \rightarrow \text{ho}\mathcal{D}$ is an eq. of cats.

fibrant objects in $s\text{Cat}_{\text{Bergner}}$ are precisely Kan-enriched cats.

([B], Thm. 4.3) — ∞ -cats

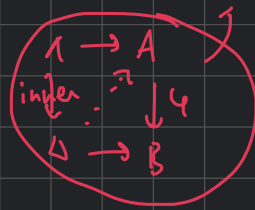
$\begin{pmatrix} \mathcal{C} \\ \downarrow \\ [0] \end{pmatrix}$ is a fibration iff $\begin{pmatrix} \text{Hom}_{\mathcal{C}}(x,y) \\ \bullet \\ \forall x,y \in \mathcal{C} \end{pmatrix}$ is a Kan cpx.

$\text{ob} = \{*\}$
 $\text{Hom}(*,*) = \Delta^0$

\Rightarrow
 \Leftarrow

$\text{fib}: f: \mathcal{C} \rightarrow \mathcal{D}$ s.t.

- $f_{x,y}: \mathcal{C}(x,y) \rightarrow \mathcal{D}(x,y)$ is a Kan fibration.

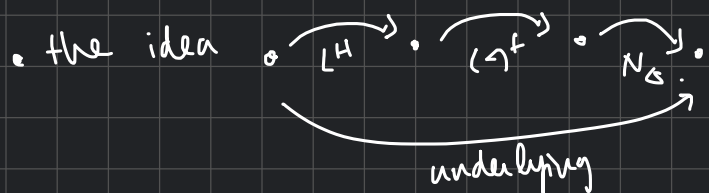


induce isofibrations on h cats.
 ?? What is this cond-saying?

Ex^∞ is what we do "locally" on Hom-ssets.

$\mathcal{C} \rightarrow \mathcal{C}^f$
 $\mathcal{C}(x,y) \rightsquigarrow \mathcal{C}^f(x,y) := Ex^\infty \mathcal{C}(x,y)$
 is a Kan-cpx.

1. model cats \rightarrow ∞ -cats.



- 2. L_H — makes a sCat
- 3. $(-)^f$ — makes a Kan-cat (∞ -cat)
- 4. N_Δ — preserves ∞ -cats (Kan-cats \mapsto q(cats))
- 5. (Quillen eq's) \rightarrow ∞ -eq's.

showing that $\text{under}(m)$ is actually an ∞ -cat.

