Twoples mtg 11/23.

$$\mathrm{RelCat} \xrightarrow{L^H} s\mathrm{Cat} \xrightarrow{(-)^f} s\mathrm{Cat} \xrightarrow{N_\Delta} q\mathrm{Cat}$$

① ~~Hammock loc'n~~

- def. of L^H.
- $\mathrm{ho}\, L^H C \simeq C[W^{-1}]$

② fibrant replacement in $s\mathrm{Cat}_{\mathrm{Bergner}}$ (ie. turning ssets ⇝ kan cpxes) hom-

③ h-c. nerve.

function $s\mathrm{Cat} \to s\mathrm{Set}$.

- intuition: ~~∞-cats as q-cats/ssets~~
- construction of N_Δ.
- $\mathfrak{C} \dashv N_\Delta$ which is a Quillen adj. $s\mathrm{Cat}_{\mathrm{Berg}} \rightleftarrows s\mathrm{Set}_{\mathrm{Joyal}}$ Q

model htpy thy of ∞-cats.

fib obj ↦ fib obj's
∞-cats (kan-enriched) = ∞-cats (q-cats)

- intuition = ∞-cats as simplicial cats. ∞-cats as Kan-enriched cats. (space) } [HTT, §1].
- construction — Ex^∞
  — fibrant replacement in $s\mathrm{Cat}_{\mathrm{Bergner}}$.

((4)) (if time) Thm: Quillen eq's ⇝ eq's of ∞-cats.

Q: In what sense does $N_\Delta C$ rep. the same htpy thy as C as a $s\mathrm{Cat}$?

(A) there's a htpy cat. function $s\mathrm{Cat} \xrightarrow{\mathrm{ho}_s} \mathrm{Cat}$

$$C \longmapsto \mathrm{ho}_s C - \mathrm{ob} = \mathrm{ob}(C), \quad \mathrm{Hom}_{\mathrm{ho}_s C}(x,y) = \pi_0 \mathrm{Hom}_C(x,y).$$

this agrees w/ the "usual" htpy cat. of qcats.

$$q\mathrm{Cat} \xrightarrow{h} \mathrm{Cat}$$
$$X \longmapsto hX - \mathrm{ob} = X_0, \quad \mathrm{Hom}_{hX}(x,y) = X_1/\sim_0$$

Turns out that these agree

$$\begin{tikzcd} s\mathrm{Cat} \arrow[r, "N_\Delta"] \arrow[dr, "\mathrm{ho}_s"'] & q\mathrm{Cat} \arrow[d, "h"] \arrow[dl, phantom, "\cong"] \\ & \mathrm{Cat} \end{tikzcd}$$

commutes.

(see here, 3.10)

[B].

Q: Ex^∞ vs. fibrant replacement?

(A): What is the model structure in question? — $sCat_{Bergner}$ — we: "D-K equivalences"

$f: \mathcal{C} \to \mathcal{D}$ in $sCat$ s.t.

- $f_{x,y}: Hom_{\mathcal{C}}(x,y)_\bullet \to Hom_{\mathcal{D}}(fx,fy)_\bullet$ is a w.e. of ssets ??
- $hof: h\mathcal{C} \to h\mathcal{D}$ is an eq. of cats.

fib: $f: \mathcal{C} \to \mathcal{D}$ s.t.

- $f_{x,y}: \mathcal{C}(x,y)_\bullet \to \mathcal{D}(x,y)_\bullet$ is a Kan fibration.

$$\begin{tikzcd} \Lambda \arrow[r] \arrow[d, "\text{incl}"'] & A \arrow[d, "\varphi"] \\ \Delta \arrow[r] \arrow[ur, dashed] & B \end{tikzcd}$$

- induce isofibrations on h cats.

?? what is this cond. saying?

fibrant objects in $sCat_{Bergner}$ are precisely Kan-enriched cats.

([B], Thm. 4.3). ∞-cats?

$\left(\begin{matrix}\mathcal{C}\\ \downarrow\\ [0]\end{matrix}\right.$ is a fibration$\left.\right)$ iff $\left(Hom_{\mathcal{C}}(x,y)_\bullet \text{ is a Kan cpx } \forall x,y \in \mathcal{C}.\right)$

$[0]$: $ob = \{*\}$, $Hom(*,*)_\bullet = \Delta^0$

$\Rightarrow$.

$\Leftarrow$

Ex^∞ is what we do "locally" on Hom-ssets.

$$\mathcal{C} \longrightarrow \mathcal{C}^f$$

$$\mathcal{C}(x,y)_\bullet \rightsquigarrow \underbrace{\mathcal{C}^f(x,y)_\bullet}_{\text{is a Kan-cpx.}} := Ex^\infty\mathcal{C}(x,y)_\bullet$$

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1. model cats + ∞-cats.
   - the idea $\bullet \xrightarrow{L^H} \bullet \xrightarrow{(-)^f} \bullet \xrightarrow{N_\Delta} \bullet$, composite: underlying

2. L^H — makes a sCat.
3. $(-)^f$ — makes a Kan-Cat (∞-cat)
4. N_Δ — preserves ∞-cats (Kan-cats ↦ q cats)
5. (Quillen eq's ⟶ ∞-eq's.)

showing that $undl(m)$ is actually an ∞-cat.

§1. Def. 1

Def. 2.

§2 Def 3.

Thm. 1.

§1. Def. 1.1

Def. 1.2

mentioned by

§2 Def. 2.1

Thm. 2.2

\colim = \operatorname{colim}.

$\varinjlim$ $\varprojlim$

\varinjlim \varprojlim.

$\lim\limits_{\Delta^n \to X}$ $\lim_{\Delta^n \to X}$.

$X, Y, \ldots$

\ldots.

$X \to Y \to \cdots \longrightarrow \cdots$

\cdots.

$X \to Y \to \ldots \longrightarrow$